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DIFFUSION MODEL OF LONGITUDINAL AGITATION IN HEAT AND MASS TRANSFER PROCESSES. 3. THIRD-LEVEL PROBLEMS

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The paper is a continuation of reports [1, 2] on a diffusion model (DM) used for describing heat and mass transfer processes. A heat exchanger (mass exchanging apparatus) is examined with two flows moving in the DM mode or in a mode of one of the limiting cases of the DM: initial displacement (ID) or ideal agitation (IA). All possible combinations are found. The problems are divided into three levels of complexity of the combinations. Derivation of formulas and solutions for determining the carrying capacities of the heat exchanger (mass exchanging apparatus) are considered as applied to the problems of the third level.

The present paper completes a series of works [1, 2] systematizing DM employment for the described heat and mass transfer processes.

The object of the analysis is a heat exchanger (mass exchanging apparatus) with two flows moving in the DM mode or in modes of its limiting cases – ideal displacement (ID) or ideal agitation (IA).

In [1] all possible combinations of these flow structures and their directions are divided into three levels of complexity. Differential equations are derived and boundary conditions are formulated. Formulas for the criterion R in the 1st-level problems are obtained [1], and a general expression for R is given. In [2] the solutions of the 2nd-level problems and limit transition from the 2nd to the 1st level are considered; an attempt is made to obtain generalized expressions for R.

Below we present an analysis of the 3rd-level problems of transfer, derive formulas for R, and study limit transitions from the 3rd to the 2nd level.

Cases in which the motion of the both flows is described by the DM are referred to the 3rd-level models. Four cases (see Table 4) are possible here. We consider two of them – forward and reverse flows (the two remaining cases are symmetric to the above).

So, a forward flow

hot (DM) →

cold (DM) →.

We write the corresponding differential equations and boundary conditions:

$$\ddot{T} - p\dot{T} - ap\,(T - t) = 0\,; \;\; x = 0\,, \;\; T' = T - \frac{1}{p}\dot{T}\,; \;\; x = 1\,, \;\; \dot{T} = 0\,; \tag{55}$$

$$\ddot{t} - q\dot{t} + bq\,(T - t) = 0\,; \;\; x = 0\,, \;\; t' = t - \frac{1}{q}\dot{t}\,; \;\; x = 1\,, \;\; \dot{t} = 0\,. \tag{56}$$

We solve the system of differential equations by the higher-order technique

$$\ddddot{T} - (p + q)\,\dddot{T} - (ap - pq + bq)\,\ddot{T} + (a + b)\,pq\,\dot{T} = 0\,. \tag{57}$$

The characteristic equation is

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$$k^4 - (p + q)\, k^3 - (ap - pq + bq)\, k^2 + (a + b)\, pqk = 0\,, \tag{58}$$

whence $k_0 = 0$, and k_1, k_2, k_3 are roots of the equation

$$k^3 - (p + q)\, k^2 - (ap - pq + bq)\, k + (a + b)\, pq = 0\,. \tag{59}$$

Then we need the properties of the roots k_1, k_2 and k_3:

$$\begin{gathered} k_1 + k_2 + k_3 = p + q\,, \\ k_1 k_2 + k_2 k_3 + k_3 k_1 = pq - ap - bq\,, \\ k_1 k_2 k_3 = -\, pq\, (a + b)\,. \end{gathered} \tag{60}$$

The solution of differential equation (57) has the form

$$T = \lambda_0 + \lambda_1 \exp(k_1 x) + \lambda_2 \exp(k_2 x) + \lambda_3 \exp(k_3 x)\,. \tag{61}$$

Having substituted T from (61) and its derivatives $\ddot{T}$, $\dot{T}$ into (55), we express t

$$t = \lambda_0 + \lambda_1 \left(-\frac{k_1^2}{ap} + \frac{k_1}{a} + 1 \right) \exp(k_1 x) + \lambda_2 \left(-\frac{k_2^2}{ap} + \frac{k_2}{a} + 1 \right) \times$$

$$\times \exp(k_2 x) + \lambda_3 \left(-\frac{k_3^2}{ap} + \frac{k_3}{a} + 1 \right) \exp(k_3 x)\,.$$

Using the boundary conditions, their combination, and the characteristic equation (59) itself, designating $T' - t' \equiv \Delta$, we obtain the following system:

$$\frac{\Delta ap}{a + b} = \lambda_1 (p - k_1) + \lambda_2 (p - k_2) + \lambda_3 (p - k_3)\,,$$

$$0 = \lambda_1 k_1 \exp k_1 + \lambda_2 k_2 \exp k_2 + \lambda_3 k_3 \exp k_3\,,$$

$$0 = \lambda_1 k_1^2 (p - k_1) \exp k_1 + \lambda_2 k_2^2 (p - k_2) \exp k_2 + \lambda_3 k_3^2 (p - k_3) \exp k_3\,.$$

The solution of this system (e.g., by the Cramer method) leads to expressions for λ_1, λ_2, λ_3:

$$\lambda_1 = \frac{\frac{\Delta ap}{a + b} \left(k_2 k_3^2 (p - k_3) \exp(k_2 + k_3) - k_3 k_2^2 (p - k_2) \exp(k_2 + k_3)\right)}{\mathrm{Zn}}\,,$$

$$\lambda_2 = \frac{\frac{\Delta ap}{a + b} \left(k_3 k_1^2 (p - k_1) \exp(k_3 + k_1) - k_1 k_3^2 (p - k_3) \exp(k_3 + k_1)\right)}{\mathrm{Zn}}\,,$$

$$\lambda_3 = \frac{\frac{\Delta ap}{a + b} \left(k_1 k_2^2 (p - k_2) \exp(k_1 + k_2) - k_2 k_1^2 (p - k_1) \exp(k_1 + k_2)\right)}{\mathrm{Zn}}\,,$$

where the denominator is

$$\mathrm{Zn} = (p - k_1) \left(k_2 k_3^2 (p - k_3) \exp(k_2 + k_3) - k_3 k_2^2 (p - k_2) \exp(k_2 + k_3)\right) +$$

$$+ (p - k_2) \left(k_3 k_1^2 (p - k_1) \exp(k_3 + k_1) - k_1 k_3^2 (p - k_3) \exp(k_3 + k_1)\right) +$$

$$+ (p - k_3)\left(k_1 k_2^2 (p - k_2) \exp (k_1 + k_2) - k_2 k_1^2 (p - k_1) \exp (k_1 + k_2)\right).$$

As a result, in the forward flow process of heat transfer the heat flux from the hot agent to the cold agent is written as

$$Q = G_1 C_1 (T' - T'') = G_1 C_1 \left(T_{x=0} - \frac{1}{p} \dot{T}_{x=0} - T_{x=1} \right) =$$

$$= G_1 C_1 \left(\lambda_1 \left(1 - \frac{k_1}{p} - \exp k_1 \right) + \lambda_2 \left(1 - \frac{k_2}{p} - \exp k_2 \right) + \lambda_3 \left(1 - \frac{k_3}{p} - \exp k_3 \right) \right).$$

We introduce the criterion $R = Q/(G_1 C_1 \Delta a) = Q/\Delta KF$, which is the ratio of the carrying capacities of the process as a whole and the stage of surface transfer. Then, using the properties of roots (60) and multiplying the fraction by exp $(-k_1 - k_2 - k_3)$, we obtain after transformations

$$R = \frac{1}{a+b} \left[1 - \left\{ pq \left(k_2 k_3 (k_2 - k_3) + (k_3 k_1 (k_3 - k_1) + k_1 k_2 (k_1 - k_2)\right) \right\} \times \right.$$
$$\times \left\{ (p - k_1)(q - k_1) k_2 k_3 (k_2 - k_3) \exp (- k_1) + (p - k_2) \times \right.$$
$$\left. \left. \times (q - k_2) k_3 k_1 (k_3 - k_1) \exp (- k_2) + (p - k_3)(q - k_3) k_1 k_2 (k_1 - k_2) \exp (- k_3) \right\}^{-1} \right]. \quad (62)$$

We consider limit transitions (62). When $q \to \infty$, the transition

$$\begin{matrix} \text{hot (DM)} \to \\ \text{cold (DM)} \to \end{matrix} \Rightarrow \begin{matrix} \text{hot (DM)} \to \\ \text{cold (ID)} \to . \end{matrix}$$

must hold. To realize this we divide the characteristic equation (58) by q

$$\frac{k^3}{q} - \left(\frac{p}{q} + 1 \right) k^2 + \left(p - \frac{ap}{q} - b \right) k + p (a + b) = 0;$$

now we have $k_3 \to q + b$ for $q \to \infty$ and roots k_1 and k_2 are found from the quadratic equation

$$k^2 - (p - b) k - p (a + b) = 0.$$

Under the considered conditions at $k_3 = q + b$ it is obvious that $p - k_3 \to -q$; $q - k_3 \to -b$; $q - k_2 \to q$; $k_2 k_3 \to q k_2$; $k_3 k_1 \to q k_1$; $q - k_1 \to q$; $k_2 - k_3 \to -q$; $k_3 - k_1 \to q$. Then Eq. (62) takes the form

$$R = \frac{1}{a+b} \left(1 - \frac{p (k_1 - k_2)}{k_1 (p - k_2) \exp (- k_2) - k_2 (p - k_1) \exp (- k_1)} \right).$$

Using the properties of roots (60) we finally obtain

$$R = \frac{1}{a+b} \left(1 - \frac{k_1 - k_2}{(a + b + k_1) \exp (- k_2) - (a + b + k_2) \exp (- k_1)} \right),$$

which is in full agreement with formula (28) obtained in [2] for the corresponding 2nd-level problem.

When $q \to 0$, the transition

$$\begin{matrix} \text{hot (DM)} \to \\ \text{cold (DM)} \to \end{matrix} \Rightarrow \begin{matrix} \text{hot (DM)} \to \\ \text{cold (IA)} \leftrightarrow . \end{matrix}$$

must hold.

For $q \to 0$, $k_3 \to q(1 + b/a)$, and roots k_1 and k_2 are found from the quadratic equation

$$k^2 - pk - ap = 0\,.$$

Under the considered conditions at $k_3 = q(1 + b/a)$ it is obvious that $p - k_3 \to p$; $q - k_3 \to -qb/a$; $k_2 - k_3 \to k_2$; $k_3 - k_1 \to k_1$; $\exp(-k_3) \to 1$.

Then Eq. (62) takes the form

$$R = \frac{1}{a+b}\left(1 - \frac{ap\,(k_1 - k_2)}{(p - k_2)\,k_1\,(a+b)\exp(-k_2) - (p - k_1)\,k_2\,(a+b)\exp(-k_1) - pb\,(k_1 - k_2)}\right).$$

Using the properties of roots (60), we obtain

$$R = \frac{(k_1 + a)\exp(-k_2) - (k_2 + a)\exp(-k_1) - (k_1 - k_2)}{(a+b)\,((k_1 + a)\exp(-k_2) - (k_2 + a)\exp(-k_1)) - b\,(k_1 - k_2)}.$$

After simple transformations we have in final form

$$R = \frac{\frac{1}{a}\left(1 - \frac{k_1 - k_2}{(a + k_1)\exp(-k_2) - (a + k_2)\exp(-k_1)}\right)}{1 + \frac{b}{a}\left(1 - \frac{k_1 - k_2}{(a + k_1)\exp(-k_2) - (a + k_2)\exp(-k_1)}\right)},$$

which is in full agreement with formula (29) obtained in [2] for the corresponding 2nd-level problem.

Similarly we can consider limit transitions for the hot agent: when $p \to \infty$ and $p \to 0$:

$$\text{when}\quad p \to \infty \quad \left\{\begin{array}{l}\text{hot (DM)} \rightarrow \\ \text{cold (DM)} \rightarrow\end{array}\right. \Longrightarrow \left\{\begin{array}{l}\text{hot (ID)} \rightarrow \\ \text{cold (DM)} \rightarrow\end{array}\right.$$

$$\text{when}\quad p \to 0 \quad \left\{\begin{array}{l}\text{hot (DM)} \rightarrow \\ \text{cold (DM)} \rightarrow\end{array}\right. \Longrightarrow \left\{\begin{array}{l}\text{hot (IA)} \leftrightarrow \\ \text{cold (DM)} \rightarrow\end{array}\right..$$

The conducted analysis shows that for the case of forward flow the formula for the DM 3rd-level problem in the limiting cases naturally passes over to the corresponding formulas of the 2nd-level problems; by the way, this indirectly indicates the correctness of the analysis and the formula.

We now consider a reverse flow (3rd-level problems)

hot (DM) →

cold (DM) ←.

The groundwork for derivation is similar to that adopted for a forward flow. We write the corresponding [1, Table 1] differential equations and boundary conditions

$$\ddot{T} - p\dot{T} - ap\,(T - t) = 0\,;\quad x = 0\,,\quad T' = T - \frac{1}{p}\dot{T}\,;\quad x = 1\,,\quad \dot{T} = 0\,; \tag{63}$$

$$\ddot{t} + q\dot{t} + bq\,(T - t) = 0\,;\quad x = 0\,,\quad \dot{t} = 0\,;\quad x = 1\,,\quad t' = t + \frac{1}{q}\dot{t}\,. \tag{64}$$

The system of differential equations is solved by the higher-order technique

$$\dddot{T} - (p - q)\,\ddot{T} - (ap + pq + bq)\,\dot{T} - (a - b)\,pq\,T = 0\,. \tag{65}$$

The characteristic equation is

$$k^4 - (p - q)\,k^3 - (ap + pq + bq)\,k^2 - (a - b)\,pqk = 0\,. \tag{66}$$

The roots are $k_0 = 0$, k_1, k_2, k_3 from the equation

$$k^3 - (p - q)\,k^2 - (ap + pq + bq)\,k - (a - b)\,pq = 0\,. \tag{67}$$

The properties of roots k_1, k_2, k_3 are

$$\begin{aligned} k_1 + k_2 + k_3 &= p - q\,, \\ k_1k_2 + k_2k_3 + k_3k_1 &= -\,(ap + pq + bq)\,, \\ k_1k_2k_3 &= pq\,(a - b)\,. \end{aligned} \tag{68}$$

The solution of differential equation (65) has the form

$$T = \lambda_0 + \lambda_1 \exp(k_1x) + \lambda_2 \exp(k_2x) + \lambda_3 \exp(k_3x)\,. \tag{69}$$

Having substituted T from (69) and its derivatives $\ddot{T}$, $\dot{T}$ into (63), we express t:

$$t = \lambda_0 + \lambda_1\left(-\frac{k_1^2}{ap} + \frac{k_1}{a} + 1\right)\exp(k_1x) + \lambda_2\left(-\frac{k_2^2}{ap} + \frac{k_2}{a} + 1\right)\exp(k_2x) + \lambda_3\left(-\frac{k_3^2}{ap} + \frac{k_3}{a} + 1\right)\exp(k_3x)\,.$$

Using the boundary conditions and their combination, designating $T' - t' \equiv \Delta$, we obtain the following system

$$\begin{aligned} \Delta ap &= \lambda_1\,(a\,(p - k_1) - b\,(p - k_1)\exp k_1) + \lambda_2\,(a\,(p - k_2) - b\,(p - k_2)\exp k_2) + \\ &\quad + \lambda_3\,(a\,(p - k_3) - b\,(p - k_3)\exp k_3)\,, \\ 0 &= \lambda_1k_1\exp k_1 + \lambda_2k_2\exp k_2 + \lambda_3k_3\exp k_3\,, \\ 0 &= \lambda_1\,(k_1\,(k_1 - p - b) - p\,(a - b)) + \lambda_2\,(k_2\,(k_2 - p - b) - p\,(a - b)) + \\ &\quad + \lambda_3\,(k_3\,(k_3 - p - b) - p\,(a - b))\,. \end{aligned}$$

The solution of this system yields

$$\lambda_1 = \frac{\Delta ap\,(k_2\exp k_2\,(k_3\,(-\,b - p + k_3) - p\,(a - b)) - k_3\exp k_3\,(k_2\,(-\,b - p + k_2) - p\,(a - b)))}{\mathrm{Zn}}\,,$$

$$\lambda_2 = \frac{\Delta ap\,(k_3\exp k_3\,(k_1\,(-\,b - p + k_1) - p\,(a - b)) - k_1\exp k_1\,(k_3\,(-\,b - p + k_3) - p\,(a - b)))}{\mathrm{Zn}}\,,$$

$$\lambda_3 = \frac{\Delta ap\,(k_1\exp k_1\,(k_2\,(-\,b - p + k_2) - p\,(a - b)) - k_2\exp k_2\,(k_1\,(-\,b - p + k_1) - p\,(a - b)))}{\mathrm{Zn}}\,,$$

where the denominator is

$$\begin{aligned} \mathrm{Zn} &= ak_3\,[(p - k_2)\,(k_1\,(-\,b - p + k_1) - p\,(a - b)) - (p - k_1)\,(k_2\,(-\,b - p + k_2) - p\,(a - b))\,]\exp k_3 + \\ &+ ak_1\,[(p - k_3)\,(k_2\,(-\,b - p + k_2) - p\,(a - b)) - (p - k_2)\,(k_3\,(-\,b - p + k_3) - p\,(a - b))\,]\exp k_1 + \\ &+ ak_2\,[(p - k_1)\,(k_3\,(-\,b - p + k_3) - p\,(a - b)) - (p - k_3)\,(k_1\,(-\,b - p + k_1) - p\,(a - b))\,]\exp k_2 + \\ &+ b\,(-\,(p - k_1)\,k_2 + (p - k_2)\,k_1)\,(k_3\,(-\,b - p + k_3) - p\,(a - b))\exp(k_1 + k_2) + \end{aligned}$$

$$+ b\,(-(p-k_2)\,k_3 + (p-k_3)\,k_2)\,(k_1\,(-b-p+k_1) - p\,(a-b))\exp(k_2+k_3) +$$

$$+ b\,(-(p-k_3)\,k_1 + (p-k_1)\,k_3)\,(k_2\,(-b-p+k_2) - p\,(a-b))\exp(k_3+k_1)\,.$$

As a result, in the reverse flow process of heat transfer the heat flux from the hot agent to the cold agent is expressed as

$$Q = G_1C_1\,(T' - T'') = G_1C_1\left(T_{x=0} - \frac{1}{p}\,\dot{T}_{x=0} - T_{x=1}\right) =$$

$$= G_1C_1\left(\lambda_1\left(1 - \frac{k_1}{p} - \exp k_1\right) + \lambda_2\left(1 - \frac{k_2}{p} - \exp k_2\right) + \lambda_3\left(1 - \frac{k_3}{p} - \exp k_3\right)\right).$$

Introducing the criterion $R = Q/(G_1C_1\Delta a) = Q/(\Delta KF)$ and using the properties of roots (68), we obtain

$$R = \Big\{[((k_2-k_3)\,(k_1\,(a+q+k_1) + q\,(a-b)))\exp k_1 + ((k_3-k_1)\,(k_2\,(a+q+k_2) + q\,(a-b)))\exp k_2 +$$

$$+ ((k_1-k_2)\,(k_3\,(a+q+k_3) + q\,(a-b)))\exp k_3]\exp(-p) - [((k_2-k_3)\,(k_1\,(-b-p+k_1) -$$

$$- p\,(a-b)))\exp(-k_1) + ((k_3-k_1)\,(k_2\,(-b-p+k_2) - p\,(a-b)))\exp(-k_2) +$$

$$+ ((k_1-k_2)\,(k_3\,(-b-p+k_3) - p\,(a-b)))\exp(-k_3)]\exp(-q)\Big\} \times$$

$$\times \Big\{a\,[((k_2-k_3)\,(k_1\,(a+q+k_1) + q\,(a-b)))\exp k_1 + ((k_3-k_1)\,(k_2\,(a+q+k_2) +$$

$$+ q\,(a-b)))\exp k_2 + ((k_1-k_2)\,(k_3\,(a+q+k_3) + q\,(a-b)))\exp k_3]\exp(-p) -$$

$$- b\,[((k_2-k_3)\,(k_1\,(-b-p+k_1) - p\,(a-b)))\exp(-k_1) + ((k_3-k_1)\,(k_2\,(-b-p+k_2) -$$

$$- p\,(a-b)))\exp(-k_2) + ((k_1-k_2)\,(k_3\,(-b-p+k_3) - p\,(a-b)))\exp(-k_3)]\exp(-q)\Big\}^{-1}. \quad (70)$$

In a shorter presentation

$$R = \frac{G - H}{aG - bH},$$

where

$$G = \exp(-p)\sum_{i=1}^{3} C_i \exp k_i\,;\quad H = \exp(-q)\sum_{i=1}^{3} D_i \exp(-k_i)\,;$$

$$C_1 = (k_2-k_3)\,((a+q+k_1)\,k_1 + q\,(a-b))\,;$$

$$C_2 = (k_3-k_1)\,((a+q+k_2)\,k_2 + q\,(a-b))\,;$$

$$C_3 = (k_1-k_2)\,((a+q+k_3)\,k_3 + q\,(a-b))\,;$$

$$D_1 = (k_2-k_3)\,((-b-p+k_1)\,k_1 - p\,(a-b))\,;$$

$$D_2 = (k_3-k_1)\,((-b-p+k_2)\,k_2 - p\,(a-b))\,;$$

$$D_3 = (k_1-k_2)\,((-b-p+k_3)\,k_3 - p\,(a-b))\,.$$

We consider the limit transition of formula (70). When $q \to \infty$ the transition

$$\begin{matrix}\text{hot (DM)} \rightarrow \\ \text{cold (DM)} \leftarrow\end{matrix} \Rightarrow \begin{matrix}\text{hot (DM)} \rightarrow \\ \text{cold (ID)} \leftarrow .\end{matrix}$$

must hold.

To realize the transition we divide characteristic equation (66) by q

$$\frac{k^3}{q} - \left(\frac{p}{q} - 1\right) k^2 - \left(p + \frac{ap}{q} + b\right) k - p\,(a - b) = 0\,.$$

For $q \to \infty$ we have $k_3 \to -(q + b)$, and roots k_1 and k_2 are found from the quadratic equation

$$k^2 - (p + b)\, k - p\,(a - b) = 0\,.$$

We substitute $k_3 = -(q + b)$ into Eq. (70) allowing for the fact that in this case $C_1 \to q^2(a - b + k_1)$; $C_2 \to -q^2(a - b + k_2)$; $C_3 \to 0$; $D_1 \to 0$; $D_2 \to 0$; $D_3 \to q^2(k_1 - k_2)$.

Then Eq. (70) takes the form

$$R = \frac{(a - b + k_2)\exp(-k_1) - (a - b + k_1)\exp(-k_2) + (k_1 - k_2)}{a\,((a - b + k_2)\exp(-k_1) - (a - b + k_1)\exp(-k_2)) + b\,(k_1 - k_2)}\,.$$

Finally

$$R = \frac{\dfrac{1}{a - b}\left(1 - \dfrac{k_1 - k_2}{(a - b + k_1)\exp(-k_2) - (a - b + k_2)\exp(-k_1)}\right)}{1 + \dfrac{b}{a - b}\left(1 - \dfrac{k_1 - k_2}{(a - b + k_1)\exp(-k_2) - (a - b + k_2)\exp(-k_1)}\right)}\,,$$

which is in full agreement with formula (30) obtained in [2] for the corresponding 2nd-level problem.

For $q \to 0$ the transition

$$\begin{matrix}\text{hot (DM)} \rightarrow \\ \text{cold (DM)} \leftarrow\end{matrix} \Rightarrow \begin{matrix}\text{hot (DM)} \rightarrow \\ \text{cold (IA)} \leftrightarrow .\end{matrix}$$

must hold.

When $q \to 0$ we have $k_3 \to (a - b)q/a$ and roots k_1 and k_2 are found from the quadratic equation

$$k^2 - pk - ap = 0\,.$$

Here the parameters of (70) are the following: $C_1 \to k_1k_2(a + k_1)$; $C_2 \to -k_1k_2(a + k_2)$; $C_3 \to 0$; $D_1 \to k_2((-b - p + k_1)k_1 - p(a - b))$; $D_2 \to -k_1((-b - p + k_2)k_2 - p(a - b))$; $D_3 \to -(k_1 - k_2)p(a - b)$. After substitution of these values, Eq. (70) takes the form

$$R = \frac{\dfrac{1}{a}\left(1 - \dfrac{k_1 - k_2}{(a + k_1)\exp(-k_2) - (a + k_2)\exp(-k_1)}\right)}{1 + \dfrac{b}{a}\left(1 - \dfrac{k_1 - k_2}{(a + k_1)\exp(-k_2) - (a + k_2)\exp(-k_1)}\right)}\,,$$

which repeats formula (29) obtained in [2] for the corresponding 2nd-level problem.

Similarly we can consider the limiting cases for the hot agent: when $p \to \infty$ and $p \to 0$

$$\text{when}\quad p \to \infty \left\{\begin{matrix}\text{hot (DM)} \rightarrow \\ \text{cold (DM)} \leftarrow\end{matrix}\right. \Rightarrow \left\{\begin{matrix}\text{hot (ID)} \rightarrow \\ \text{cold (DM)} \leftarrow\end{matrix}\right.$$

TABLE 4. Levels of Complexity of Problems and Numbers of Computational Relations

Hot	Cold				
	ID → m = 1	DM → m = 1	IA ↔ m = 0	DM ← m = −1	ID ← m = −1
ID → l = 1	6*	34**	9*	37**	12*
DM → l = 1	28**	62	29**	70	30**
IA ↔ l = 0	7*	35**	10*	38**	13*
DM ← l = −1	31**	70	32**	62	33**
ID ← l = −1	8*	36**	11*	39**	14*

Note: * Formulas (6)-(14) see in [1]; ** formulas (28)-(39) see in [2].

$$\text{when } p \to 0 \begin{cases} \text{hot (DM)} \rightarrow \\ \text{cold (DM)} \leftarrow \end{cases} \Rightarrow \begin{cases} \text{hot (IA)} \leftrightarrow \\ \text{cold (DM)} \leftarrow . \end{cases}$$

The conducted analysis shows that in the case of a reverse flow the formula for the DM 3rd-level problems in the limiting cases naturally passes over to the corresponding formulas of the 2nd-level problems.

Formulas for the 3rd-level problems are listed in Table 4 in accordance with the level and sign variables.

We note that we did not succeed in reducing the 3rd-level problems to one general formula.

Thus, the present work systematizes, to a certain extent, the description of heat and mass transfer processes at the level of the DM of flows. The correctness of the obtained formulas is confirmed by their limit transitions to the lower level.

Quantitative estimates of the carrying capacity of the process show that the value Q/Δ for a heat exchanger with flows moving in the DM mode, as should be expected, is intermediate from Q/Δ for heat exchangers where both flows move in the IA mode or in the ID mode. Calculation by the given formulas also illustrates the effect of agitation on Q/Δ: Q/Δ grows with $p(q)$.

It is expedient to continue the study for composing general formulas of a higher-order level that provide limit transitions to lower levels.

It is also of interest to use fractional values of sign functions for solving higher-level problems on the basis of simpler formulas for lower-level problems.

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